Introduction to block structured nonlinear systems

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Identificazione di sistemi nonlineari

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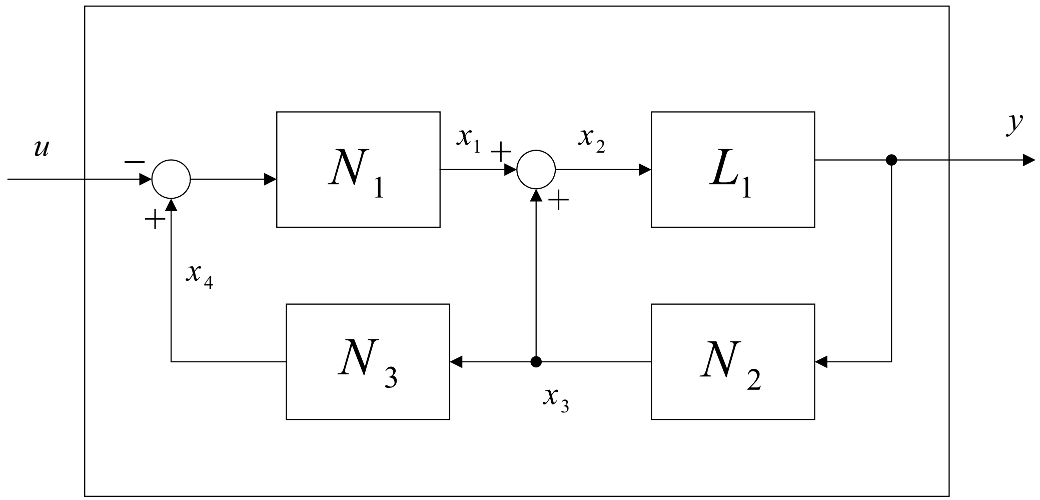
Definition

Block structured models are nonlinear systems made up of a number of interconnected linear and nonlinear subsystems



- N_1, N_2, N_3: nonlinear subsystems
- L_1: linear subsystem

Main attractive features

- Ability to embed process structure knowledge

$\Rightarrow$ More accurate description of process behaviour

$\Rightarrow$ Identification of high-order nonlinear systems (hard problem) reduced to identification of lower order subsystems and their interactions.

$$\Downarrow$$

<u>Improved identification accuracy</u>

Identification of block-structured systems

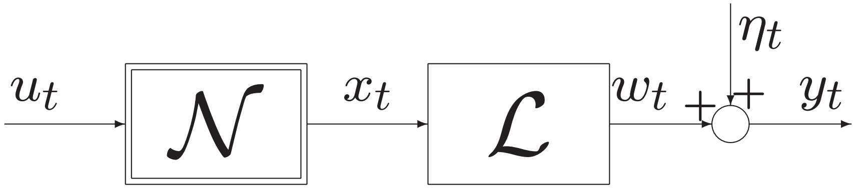
- **Aim**: find a model for each subsystems (e.g. N_1, N_2, N_3, L_1)

- **Main Constraint**: inner signals (e.g. x_1, x_2, x_3, x_4) are *not measurable*
  $\Rightarrow$ identification of the subsystems based on:
  - a set of *prior assumptions* on the system to be identified
  - a set of (noise corrupted) *measurements* of the input and output signals u and y.

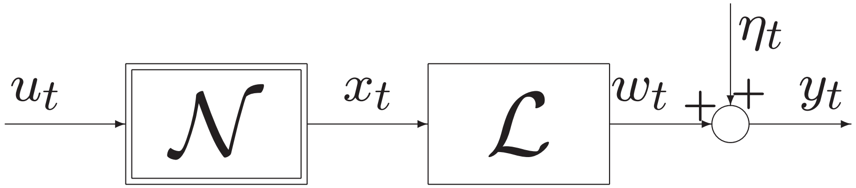
Some basic nonlinear block structured models

In this lesson we will (mainly) focus on the the following two classes of block-oriented non-linear models:

Hammerstein systems



Wiener systems



- $\mathcal{N}$: static nonlinearity
- $\mathcal{L}$: linear dynamic subsystem
- x_t: inner signal not measurable
- η_t: output measurement noise

Hammerstein and Wiener systems: applications

- Hammerstein systems are useful to describe (essentialy) linear dynamic process driven by a nonlinear actuator (with negligible dynamics)

- Wiener systems are useful to describe (essentially) linear dynamic process equipped with nonlinear sensor (with negligible dynamics)

- Despite their simplicity, such models have been successfully used in many engineering fields (signal processing, identification of biological systems, modeling of distillation columns, modeling of hydraulic actuators, etc.)

- Thanks to their simple structure Hammerstein and Wiener systems are quite attractive from the user point of view $\Rightarrow$ *often used to approximate more complex nonlinear systems*

Block-structured systems as approximated models

- Nonlinear block-structured system can be effectively used to approximate more complex nonlinear system.

- More precisely block-structured system can be effectively used to approximate any process in the class of *Fading Memory Nonlinear systems*.

Notion of *Fading Memory* (Intuition)

Roughly speaking a nonlinear dynamic system has fading memory if two input signals which are close in the recent past, but not necessarily close in the remote past, yield present outputs which are close.

Block-structured systems as approximated models:

Main results

Result 1 (S. Boyd, L. Chua, *IEEE Trans. on Circuits and Systems 1985*)

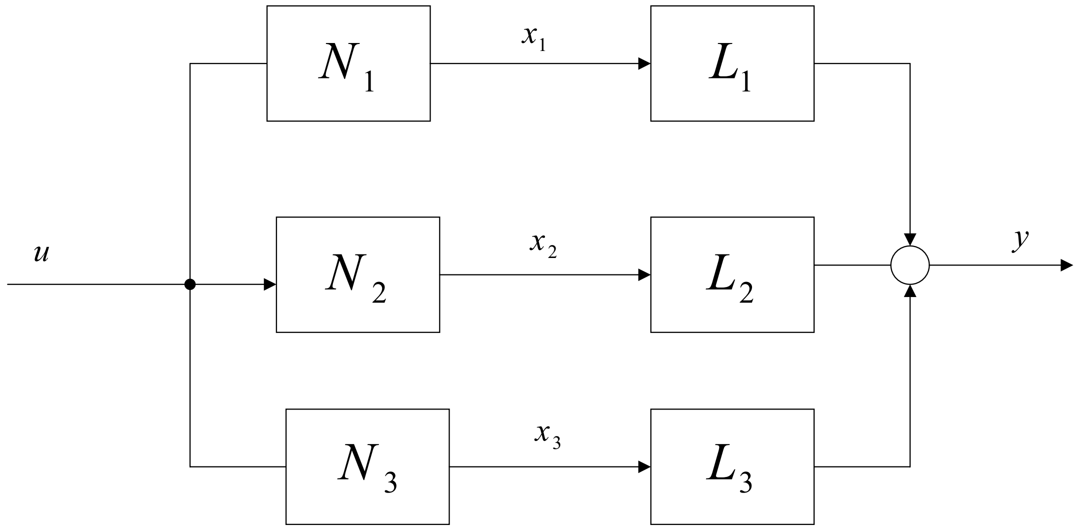
Any nonlinear system which has a *Fading Memory* can be approximated to an arbitrary degree of accuracy by a finite Volterra functional expansion.

Result 2 (M. Korenberg, *Annals of Biomed. Eng. 1991*)

Any nonlinear system which has a finite Volterra functional expansion can be approximated to an arbitrary degree of accuracy (in the mean square sense) by a (finite) sum of Wiener systems.

Parallel cascade nonlinear systems

The sum of a finite number of Wiener systems is usually called *Parallel cascade structure*



- $N_1, N_2, N_3, ..$ are nonlinear static blocks
- $L_1, L_2, L_3, ..$ are linear dynamic systems
- Parallel cascade structure are used to model (fading memory) nonlinear systems of unknown structure
- Blocks N_i, L_i and signals x_i do not have physical meaning

Parallel cascade nonlinear systems: Identification

The problem of building (identifying) an *approximated parallel cascade model* of a nonlinear system (of unknown structure) can be reduced to the identification of n Wiener systems

Algorithm (basic idea)

1. Stimulate the nonlinear process with a (proper) input $u(t)$ and collect measurements of output $\tilde{y}(t)$

2. Use experimental data $u(t), \tilde{y}(t)$ to identify the Wiener model (N_1, L_1) which provide the best fit of the data (first branch of the parallel structure)

3. Compute the error $e(t) = \tilde{y}(t) - y(t)$ between the output of the process ($\tilde{y}$) and the output of the parallel cascade model (y)

4. Use signals $u(t)$ and $e(t)$ to identify a new Wiener model which provide the best fit of such data (added to the parallel structure as a new branch)

5. Repeat from step 3 until the desired accuracy is obtained

Hammerstein and Wiener systems: Identification

Statistical framework:

Many approaches have been proposed (see list of references for details):

- Iterative approach
- Overparameterization method
- Separable least-squares approach
- Frequency domain approach
- Stochastic method (kernel approach)
- Subspace approach

Focus of this lesson $\longrightarrow$ **Set-membership framework**

References

Survey

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Identication algorithm for Hammerstein and Wiener systems (statistical framework)

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Approximation properties of block-structured models

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